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LETTER TO THE EDITOR

The interacting boson–fermion model and the labelling of states in SO(5)

Joris Van der Jeugt†

Seminarie voor Wiskundige Natuurkunde, Rijksuniversiteit Gent, Krijgslaan 281-S9, B9000 Gent, Belgium

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Abstract. A new labelling is presented for basis states of the non-symmetric SO(5) representations $\{\tau_1, \tau_2\} = \{v+\frac{1}{2}, \frac{1}{2}\}$ appearing in the interacting boson-fermion model. In contrast with a previously introduced label, the newly proposed labels have a physical interpretation.

Symmetric representations $\{v, 0\}$ of SO(5) have been used extensively in classification problems for quadrupole phonon states in the collective model for the nucleus (Bohr 1952, Bohr and Mottelson 1953), as well as for states in the SO(5) limit of the interacting boson model (Arima and Iachello 1976, 1978, 1979). In these models, the representations $\{v, 0\}$, constructed out of couplings of d-bosons, are decomposed according to SO(3). The SO(3) labels l and m are the total angular momentum, and its projection, of a basis state. However, the reduction from SO(5) to SO(3) is not fully reducible: in general there are two missing labels, and for the symmetric representations $\{v, 0\}$ there is one missing label. The latter problem has been solved by Chacón *et al* (1976) and Chacón and Moshinsky (1977): it is shown that a missing label ν can be introduced, such that the basis states of $\{v, 0\}$ are described by

$$|v, \nu, l, m\rangle, \tag{1}$$

where $\nu = 0, 1, 2, \ldots, [v/3]$,

$$v = 3\nu + K,$$

and $l = 2K, 2L-2, 2K-3, \ldots, K$.

Besides l and m, also v and ν have a physical meaning: v is the seniority, and ν is the number of (traceless) boson-triplets coupled to angular momentum 0. The basis states (1) can be expressed in terms of elementary permissible diagrams (EPD) $[v, l]$ (Chacón *et al* 1976):

$$|v, \nu, l, m=l\rangle = [1,2]^{l-v+3\nu}[2,2]^{(2v-l)/2-3\nu}[3,0]^{\nu}|0\rangle, \qquad \text{for } l \text{ even}; \tag{2a}$$

$$|v, \nu, l, m=l\rangle = [1,2]^{l-v+3\nu}[2,2]^{(2v-3-l)/2-3\nu}[3,3][3,0]^{\nu}|0\rangle, \qquad \text{for } l \text{ odd}. \tag{2b}$$

In fact, the labelling (1) as well as the expressions (2) follow from the branching rule generating function (GF) $G(V, L)$ for symmetric representations $\{v, 0\}$ of SO(5) to

† Research Assistant NFWO (Belgium).

representations (l) of SO(3):

$$G(V, L)=\frac{1+V^3L^3}{(1-V^3)(1-VL^2)(1-V^2L^2)}. \tag{3}$$

Indeed, the expansion of (3) can be written as

$$G(V, L)=\left(\sum_{\nu=0}^{\infty} V^{3\nu}\right)\left(\sum_{K=0}^{\infty} V^K(L^{2K}+L^{2K-2}+L^{2K-3}+\ldots+L^K\right), \tag{4}$$

from which (1) is apparent. On the other hand, the first term in the numerator of (3) gives rise to equations of the form

$$V^vL^l=(V^3)^\nu(VL^2)^a(V^2L^2)^b, \tag{5}$$

from which the exponents $a=l-v+3\nu$ and $b=\frac{1}{2}(2v-l)-3\nu$ in $(2a)$ can be deduced; analogous remarks hold for $(2b)$.

Recently, boson-fermion symmetries have gained a lot of importance. In the interacting boson-fermion model (IBFM) (Iachello and Kuyucak 1981, Bijker and Kota 1984, Bijker and Iachello 1985), non-symmetric SO(5) representations $\{\tau_1, \tau_2\}=\{v+\frac{1}{2}, \frac{1}{2}\}$ $(v\in\mathbb{N})$ are of particular interest (Iachello 1980, Bijker and Iachello 1985). Such representations describe the basis states of a single $j=\frac{3}{2}$ fermion coupled to a number of d-bosons with seniority v. Iachello (1980) proposed the following labelling of the basis states for $\{\tau_1, \tau_2\}=\{v+\frac{1}{2}, \frac{1}{2}\}$:

$$|\tau_1, \tau_2; \nu_\Delta, j, m\rangle,$$

where

$$\nu_\Delta=0, \tfrac{1}{2}, 1, \ldots, (3\nu_\Delta\leq v), \tag{6}$$

and

$$j=2v-6\nu_\Delta+\tfrac{3}{2}, 2v-6\nu_\Delta+\tfrac{1}{2}, \ldots, v-3\nu_\Delta-\tfrac{1}{4}[1-(-1)^{2\nu_\Delta}]+\tfrac{3}{2}.$$

This labelling gives the correct branching rule to SO(3) (we use the notation SO(3) for the Lie algebra as well as for the Lie group), but ν_Δ is just a partition label with no physical meaning.

In the present letter, we will propose a new labelling for the states of $\{v+\frac{1}{2}, \frac{1}{2}\}$. This is based on the GF for the reduction of irreps $\{v+\frac{1}{2}, \frac{1}{2}\}$ to representations (l) of SO(3). First, note that the Dynkin labels of an irrep $\{\tau_1, \tau_2\}$ are given by $(a_1, a_2)=(2\tau_2, \tau_1-\tau_2)$. Hence, we must obtain the branching for irreps $(1, v)$. Therefore, we can make use of a GF by Gaskell *et al* (1978), in order to obtain:

$$G'(V, L)=\frac{L^{3/2}+VL^{1/2}+VL^{5/2}+V^2L^{3/2}}{(1-V^3)(1-VL^2)(1-V^2L^2)}. \tag{7}$$

The first term in the numerator, $V^0L^{3/2}$, corresponds to the purely fermionic states with $v=0$ and $l=\frac{3}{2}$. These can be realised as $a^+_\mu|0\rangle$ $(\mu=-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2})$, and they are the basis states of the four-dimensional irrep $(1, 0)$ of SO(5). Analogously, $VL^{1/2}$ and $VL^{5/2}$ correspond to the $l=\frac{1}{2}$ and $l=\frac{5}{2}$ states in the decomposition of the irrep $(1, 1)$, and $V^2L^{3/2}$ to the $l=\frac{3}{2}$ states in $(1, 2)$. The latter states can be obtained from coupling states of symmetric irreps $\{v, 0\}=(0, v)$ to the elementary states of $(1, 0)$, since

$$\begin{aligned}&(0, 1)\times(1, 0)\supset(1, 1),\\ &(0, 2)\times(1, 0)\supset(1, 2).\end{aligned} \tag{8}$$

The basis states of $(1, v)$ can be expressed in terms of EPD $[v, l]$ (in purely traceless bosonic operators) and $\langle v, l\rangle$ (couplings of traceless bosonic operators to the fermions a_μ^+):

$$[1,2]^{3\nu-v+l-3/2}[2,2]^{v-3\nu-l/2+3/4}[3,0]^\nu\langle 0,\tfrac{3}{2}\rangle|0\rangle, \tag{9a}$$

$$[1,2]^{3\nu-v+l+1/2}[2,2]^{v-3\nu-l/2-3/4}[3,0]^\nu\langle 1,\tfrac{1}{2}\rangle|0\rangle, \tag{9b}$$

$$[1,2]^{3\nu-v+l-3/2}[2,2]^{v-3\nu-l/2+1/4}[3,0]^\nu\langle 1,\tfrac{5}{2}\rangle|0\rangle, \tag{9c}$$

$$[1,2]^{3\nu-v+l+1/2}[2,2]^{v-3\nu-l/2-5/4}[3,0]^\nu\langle 2,\tfrac{3}{2}\rangle|0\rangle. \tag{9d}$$

Note that the multiplet with $l=\frac{5}{2}$ from $(1, 1)$ (from coupling one d-boson to the single fermion) can be seen as a 'parallel coupling' of the boson and the fermion, whereas the multiplet with $l=\frac{1}{2}$ in $(1, 1)$ is an 'anti-parallel coupling'. Also, $l=\frac{3}{2}$ from $(1, 0)$ is purely fermionic (and hence parallel coupled), and $l=\frac{3}{2}$ from $(1, 2)$ is again anti-parallel coupled. This indicates that the first and the third term in the numerator of (7) can be combined, and also the second and the fourth term. If (7) is expanded in this way, we find:

$$G'(V,L)=\left(\sum_{\nu=0}^{\infty} V^{3\nu}\right)\left(\sum_{K=0}^{\infty} V^K(L^{2K+3/2}+L^{2K+1/2}+\ldots+L^{K+3/2})\right) \tag{10a}$$

$$+\left(\sum_{\nu=0}^{\infty} V^{3\nu}\right)\left(\sum_{K=1}^{\infty} V^K(L^{2K-3/2}+L^{2K-5/2}+\ldots+L^{K-1/2})\right). \tag{10b}$$

This gives rise to the following labelling of the basis states of $(1, v)$ or $\{\tau_1, \tau_2\}=\{v+\frac{1}{2}, \frac{1}{2}\}$:

$$|\tau_1, \tau_2; \nu, \mu, j, m\rangle, \tag{11}$$

where

$$\nu=0,1,2,\ldots,v/3,\ \mu=+\text{ or }-,\ v=3\nu+K,$$

and

$$j=2K+\tfrac{3}{2}, 2K+\tfrac{1}{2},\ldots,K+\tfrac{3}{2} \qquad \text{for } \mu=+,$$

$$j=2K-\tfrac{3}{2}, 2K-\tfrac{5}{2},\ldots,K-\tfrac{1}{2} \qquad \text{for } \mu=-.$$

Hence, we have proposed a set of quantum numbers with two supplementary labels ν and μ, where μ assumes only the two values + or −. The importance of the labelling (11) is that the quantum numbers have a physical interpretation: ν still indicates the number of boson triplets coupled to angular momentum 0, and μ indicates whether the remaining bosons are parallel ($\mu=+$) or anti-parallel ($\mu=-$) coupled to the single fermion.

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